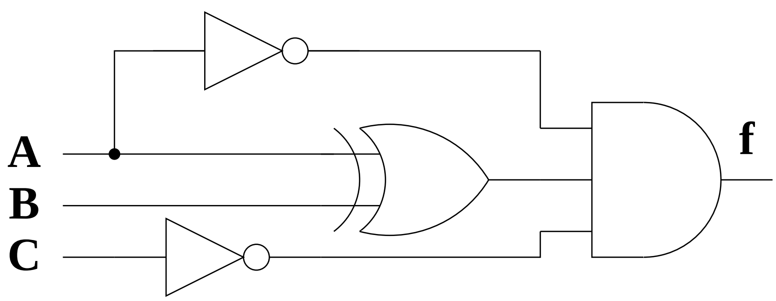
Embedded Real-Time Systems (AME 3623)
Homework 1 Solutions

February 25, 2009

Question 1

Consider the following circuit.



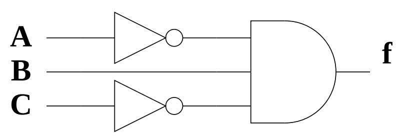
1. (10 pts) What is the corresponding truth table?

   Shortcut: f can only be true when A and C are false.

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

2. (10 pts) Show the simplified circuit (this should require very little reduction).

   There is only one row for which this circuit produces a '1'.



Question 2

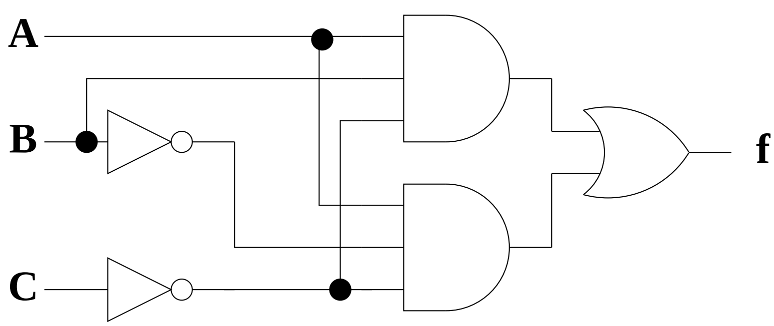
Consider the following function:

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

1. (10pts) Show the algebraic expression for the "minterm" form of the circuit (set of 3-term ANDs that are then ORed together).

   $f = A\bar{B}\bar{C} + AB\bar{C}$

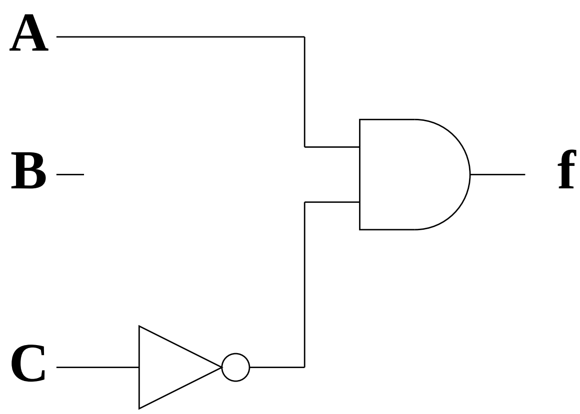
2. (10pts) Show the corresponding circuit



3. (10pts) Reduce this algebraic expression to a minimal form (note that there may be more than one correct answer). **Show each step, showing the name of the algebraic rule that you use.**

$$
\begin{aligned}
A\bar{B}\bar{C} + AB\bar{C} & \\
A(\bar{B} + B)\bar{C} &\quad \textit{Distributive Law} \\
A(1)\bar{C} &\quad \bar{X} + X = 1 \\
A\bar{C} &\quad X * 1 = X
\end{aligned}
$$

4. (10pts) Show the corresponding circuit



Question 3

Consider the following function:

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

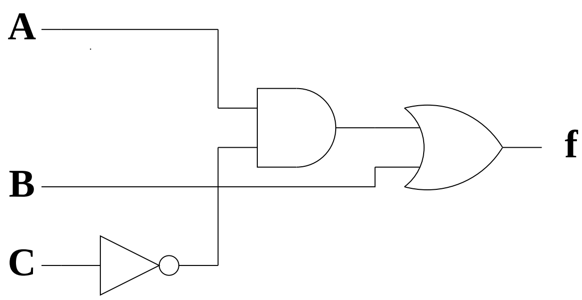
1. (10pts) Show the algebraic expression for the "minterm" form of the circuit.

   $$\bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

2. (10pts) Reduce this algebraic expression to a minimal form. Show each step, showing the name of the algebraic rule that you use.

$$\begin{aligned}
\bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + ABC & \\
\bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + AB\bar{C} + ABC & \quad X + X = X \\
\bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC + A\bar{B}\bar{C} + AB\bar{C} & \quad CommutativeLaw \\
\bar{A}B(\bar{C} + C) + AB(\bar{C} + C) + A(\bar{B} + B)\bar{C} & \quad DistributiveLaw \\
\bar{A}B(1) + AB(1) + A(1)\bar{C} & \quad X + \bar{X} = 1 \\
\bar{A}B + AB + A\bar{C} & \quad 1 * X = X \\
(\bar{A} + A)B + A\bar{C} & \quad DistributiveLaw \\
(1)B + A\bar{C} & \quad X + \bar{X} = 1 \\
B + A\bar{C} & \quad 1 * X = X
\end{aligned}$$

3. (10pts) Show the reduced circuit.



Question 4

1. (10 pts) Suppose you need a circuit to perform a NOT, but that you only have 2-input NAND gates. What would the circuit look like? Show the algebraic rules that you use. Hint: you may find it useful to tie an input to a constant 0 or 1.

   The algebraic expression for our NAND gate is:

   $C = \overline{A * B}$

   We can wire inputs A and B to anything we like. In particular, we can choose: $B = A$. This gives us:

   $C = \overline{A * A}$

   Which simplifies to (by $X = X * X$):

   $C = \overline{A}$

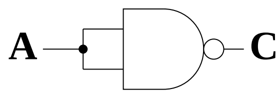
   Alternatively, we can choose $B = 1$ (i.e. a constant):

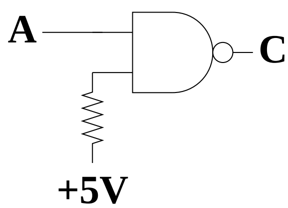
   $C = \overline{A * 1}$

   Which simplifies to (by $X = X * 1$):

   $C = \overline{A}$

The corresponding circuits are:





2. (10 pts) Suppose you need a circuit to perform a NOR between two inputs, but that you only have 2-input NAND gates. What would the circuit look like? Show the algebraic rules that you use.

   DeMorgan's Law gives us the relationship between AND and OR:

   $\overline{A+B} = \overline{A} * \overline{B}$

   The left hand side is our NOR.

   By $\overline{\overline{X}} = X$:

   $\overline{A+B} = \overline{\left(\overline{\overline{A} * \overline{B}}\right)}$

   Inside the parentheses is a NAND gate. We now just need to invert the inputs and the outputs. We already know how to do this (from above).

   The resulting circuit is: