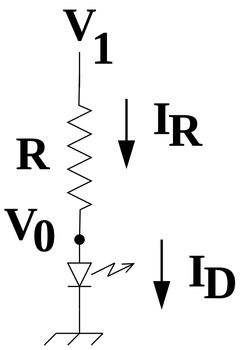
Embedded Real-Time Systems (AME 3623)
Homework 1 Solutions

February 5, 2012

Question 1

Consider the following circuit:



Assume that $V_f = 2V$ (the forward voltage of the diode).

1. (10pts) What equations are always true, no matter the state of the diode?

$$\begin{aligned} V_1 - V_0 &= RI_R \\ I_D &= I_R \end{aligned}$$

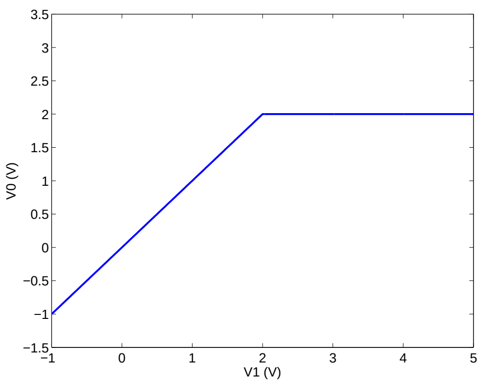
2. (15pts) At what V_1 does the diode begin to conduct current?

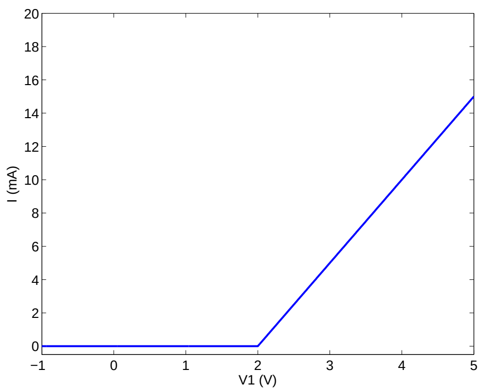
   The diode conducting current implies that: $I_D > 0$ and $V_0 - 0 = V_f$.

   "Beginning to conduct current" implies that I_D is very small. This means that the voltage drop across the resistor is very small, i.e. $V_1 \approx V_0$.

   Therefore: $V_1 \approx V_f$

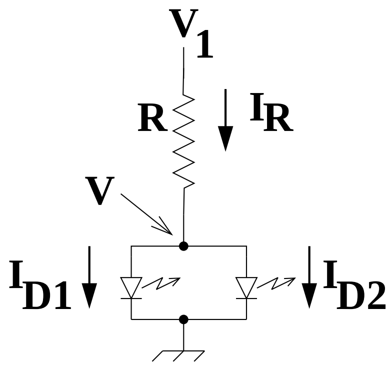
3. (10pts) Assume that $R = 200\Omega$. Draw V_0 and I_D as a function of V_1 for interesting values of V_1. (use separate figures)





Question 2

Consider the following circuit:



Assume that $V_{f1} = 2V$, $V_{f2} = 1.5V$, and $R = 1K\Omega$.

1. (10pts) What equations are always true, no matter the states of the diodes?

$$\begin{aligned} V_1 - V &= RI_R \\ I_R &= I_{D1} + I_{D2} \end{aligned}$$

2. (10pts) What are the four possible cases for the diodes?

   (a) Both on
   (b) Both off
   (c) D1 on; D2 off
   (d) D1 off; D2 on

3. (10pts) At what V_1 does some diode begin to conduct current?

   Given the next question, we know that both diodes can't be on at the same time. So, we either have D1 or D2 turning on first as we increase V_1.

   If it was D1 that turned on first (with D2 off), then we know that $V - 0 = V_{f1}$ and $I_{D1} \approx 0$. Furthermore, we know that $V - 0 < V_{f2}$. Therefore $V_1 \approx V_{f1} = 2V$, but this is a contradiction to the inequality.

If it was D2 that turned on first (with D1 off), then we know that $V - 0 = V_{f2}$ and $I_{D2} \approx 0$. Furthermore, we know that $V - 0 < V_{f1}$. Therefore $V_1 \approx V_{f2} = 1.5V$; this is consistent with the inequality.

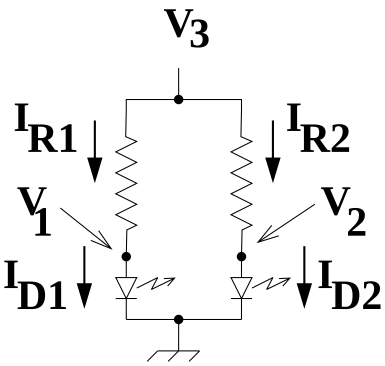
So: $V_1 = V_{f2} = 1.5V$.

4. (10pts) At what V_1 do both diodes begin to conduct current?

   Both diodes implies: $V - 0 = V_{f1}$ and $V - 0 = V_{f2}$. This is a contradiction given the forward voltages. So: this case never happens.

Question 3

Consider the following circuit:



Assume that $V_{f1} = 2V$, $V_{f2} = 1.5V$, and $R_1 = R_2 = 1K\Omega$.

1. (10pts) What equations are always true, no matter the state of the diode?

$$
\begin{aligned}
V_3 - V_1 &= R_1 I_{R1} \\
V_3 - V_2 &= R_2 I_{R2} \\
I_{R1} &= I_{D1} \\
I_{R2} &= I_{D2} \\
I &= I_{R1} + I_{R2}
\end{aligned}
$$

2. (10pts) What are the four possible cases for the diodes?
   (a) Both on
   (b) Both off
   (c) D1 on; D2 off
   (d) D1 off; D2 on

3. (10pts) At what V_3 does some diode begin to conduct current?

   Either D1 or D2 are on (or both).

If D1 is on: $I_{D1} > 0$ and $V_1 - 0 = V_{f1}$. If D1 is just turning on, then $I_{D1} \approx 0$. Then $V_3 - V_1 \approx 0$. Therefore $V_3 \approx V_{f1} = 2V$.

If D2 is on: $I_{D2} > 0$ and $V_2 - 0 = V_{f2}$. If D2 is just turning on, then $I_{D2} \approx 0$. Then $V_3 - V_2 \approx 0$. Therefore $V_3 \approx V_{f2} = 1.5V$.

Since the threshold voltage is different for the two diodes, then D2 must be the one that turns on first. This happens at $V_3 = 1.5V$.

4. (10pts) At what V_3 do both diodes begin to conduct current?

   From the previous question, V_3 must be $2V$.

5. (10pts) When $V_3 = 5V$, what are I_{R1} and I_{R2}?

   We know from the previous question that both diodes must be turned on. Therefore, $V_1 = 2V$ and $V_2 = 1.5V$.

   $I_{R1} = \frac{V_3 - V_{f1}}{I_{R1}} = \frac{3}{1000} = 3mA$

   $I_{R2} = \frac{V_3 - V_{f2}}{I_{R2}} = \frac{3.5}{1000} = 3.5mA$

   One thing to note: the two branches of the circuit are independent of one-another. If we change the forward voltage of one of the diodes or if we change one of the resistances, the other branch does not change state.

Question 4

1. (5pts) Given the binary number: 111001011. What is the decimal equivalent? Assume that this is an unsigned number. Show your work.

   $1 + 2 + 8 + 64 + 128 + 256 = 459$

2. (5pts) Assume that we interpret 11001110 a signed 8-bit number (two's complement). Is it positive or negative? What is the decimal equivalent? Show your work.

   It is negative.

   $-128 + 64 + 8 + 4 + 2 = -50$

3. (5pts) Assume that we interpret 01101100 a signed 8-bit number (two's complement). Is it positive or negative? What is the decimal equivalent? Show your work.

   It is positive.

   $64 + 32 + 8 + 4 = 108$

4. (5pts) Given the decimal number: 148. What is the binary equivalent? Show your work (all of the steps of the algorithm that we discussed in class).

value	**binary**
148	x
74	0
37	00
18	100
9	0100
4	10100
2	010100
1	0010100
0	10010100

5. (5pts) Given the decimal number: 563. What is the binary equivalent? Show your work.

value	**binary**
563	x
281	1
140	11
70	011
35	0011
17	10011
8	110011
4	0110011
2	00110011
1	000110011
0	1000110011