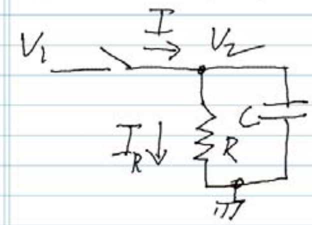
Quiz #2 1/22/13



Assume: V_1 is constant

Knowns: V_1, R, C

Unknowns: I, I_R, I_C, V_2

(A) What is always true?

$$I = I_R + I_C$$

$$V_2 - 0 = I_R R$$

$$C \frac{d(V_2 - 0)}{dt} = I_C$$

(B) What else is true when the switch is closed?

$$V_2 = V_1 \qquad + \quad \frac{dV_2}{dt} = 0$$

(C) Solve for I_R, I_C, V_2

$$V_2 = V_1$$

$$I_C = C \frac{d(V_2 - 0)}{dt} = C \frac{dV_2}{dt} = 0$$

$$I_R = \frac{V_2 - 0}{R} = \frac{V_1}{R}$$

(D) When the switch is opened at time $t=0$, what is $V(t)$ for $t>0$?

Assume $V(0) = V_1$

$$I_C = C\frac{dV_2}{dt} = -I_R = -\frac{V_2}{R}$$

$$\underbrace{\frac{dV_2}{V_2}}_{} = \underbrace{\frac{-dt}{RC}}_{}$$

Integrate:

$$\frac{-1}{RC}\int_0^T dt = \frac{-T}{RC}$$

$$\int_{V_2(0)}^{V_2(T)} \frac{dV_2}{V_2} = \ln V_2 \Big|_{V_2(0)}^{V_2(T)}$$

$$= \ln V_2(T) - \ln V_2(0)$$

$$= \ln \frac{V_2(T)}{V_2(0)} = \frac{-T}{RC}$$

$$\rightarrow \quad \frac{V_2(T)}{V_2(0)} = e^{-T/RC}$$

$$V_2(T) = V_1 e^{-T/RC}$$

Check: $V_2(T)$ at $T=0 = V_1$ ✓

$V_2(T)$ at $T=\infty = 0$ ✓

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