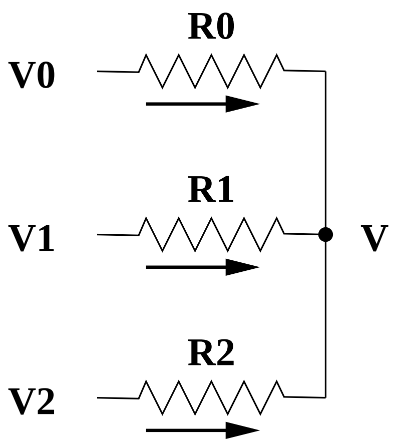
Embedded Systems (CS [45]163)
Homework 1 Solutions

February 18, 2009

Question 1

Consider the following circuit:



1. (20pts) Derive an equation for V given Vx and Rx, where $x = \{0, 1, 2\}$. Show your work.

$$\begin{aligned} V_0 - V &= R_0 I_0 \\ V_1 - V &= R_1 I_1 \\ V_2 - V &= R_2 I_2 \\ I_0 + I_1 + I_2 &= 0 \end{aligned}$$

This gives us:

$$\begin{aligned} \frac{V_0 - V}{R_0} + \frac{V_1 - V}{R_1} + \frac{V_2 - V}{R_2} &= 0 \\ \frac{\sum_{i=0}^{2} V_i/R_i}{\sum_{i=0}^{2} 1/R_i} &= V \end{aligned}$$

2. (10pts) Assume $R0 = 200$, $R1 = 100$ and $R2 = 50$. Simplify the equation for V.

$$V = \frac{\sum_{i=0}^{2} 2^i V_i}{\sum_{i=0}^{2} 2^i} = \frac{V_0+2V_1+4V_2}{1+2+4} = \frac{V_0+2V_1+4V_2}{7}$$

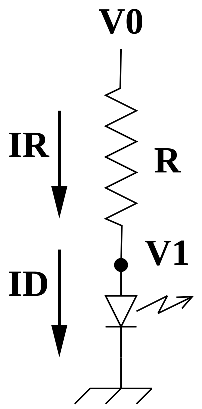
Note: you will end up with the same solution as long as the ratios of the resistances stay the same.

3. (10pts) Fill in the following table.

$V2$	$V1$	$V0$	V
0	0	0	0 V
0	0	5	5/7 V
0	5	0	10/7 V
0	5	5	15/7 V
5	0	0	20/7 V
5	0	5	25/7 V
5	5	0	30/7 V
5	5	5	5 V

Question 2

Consider the following circuit:



Assume that $V_f = 1.2V$.

1. (20pts) Derive equations for $V1$ and ID for arbitrary $V0$ and R.

   These are always true:

$$V_0 - V_1 \quad = \quad RI_R$$

$$I_D = I_R$$

First, we assume that $I_D > 0$ (i.e., that current is flowing through the diode):

This gives us:

$$V_1 - 0 = 1.2V$$

Therefore:

$$I_D = \frac{V_0 - 1.2}{R}$$

This equation is correct as long as $I_D > 0$, which means that $V_0 > 1.2$.

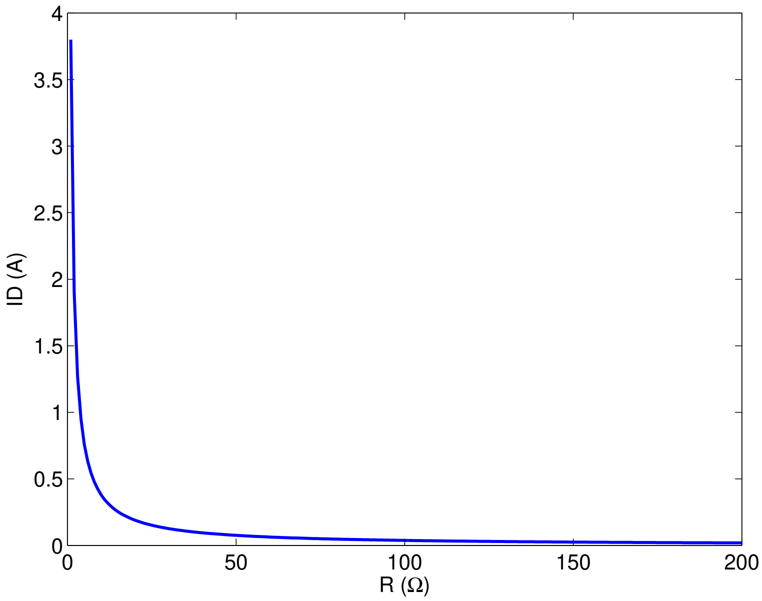
Now, let is assume that $I_D = 0$. This means that:

$$\begin{aligned} V_1 - 0 &< 1.2V \\ V_1 &< 1.2V \end{aligned}$$

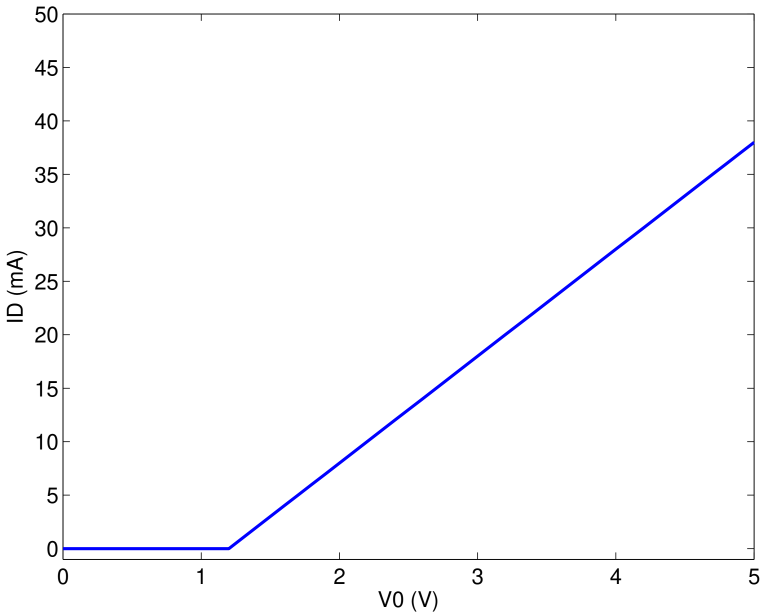
And:

$$\begin{aligned} I_R &= 0 \\ V_1 &= V_0 \end{aligned}$$

2. (10pts) Assume $V0 = 5V$. Show a plot of ID as a function of R.

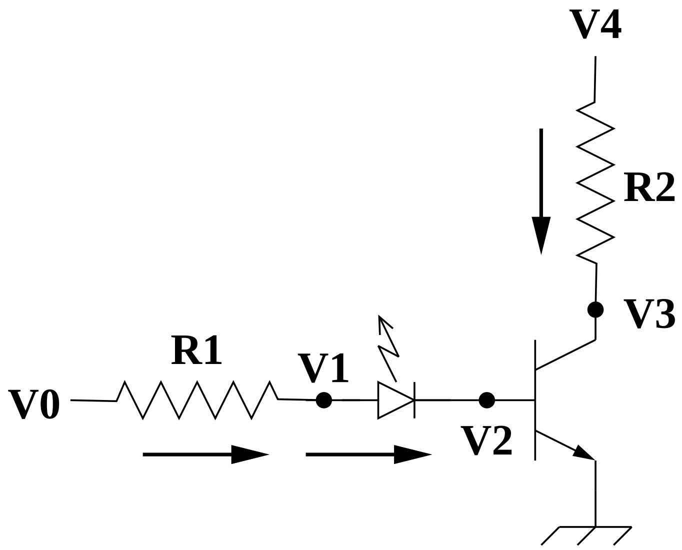


3. (10pts) Assume $R = 100\Omega$. Show a plot of ID as a function of $V0$. Be sure to include all interesting values of $V0$.



Question 3

Consider the following circuit:



Assume $V_{fD} = 1V$, $V_{fT} = 0.5V$, $g = 100$, $R1 = 1K\Omega$, $R2 = 100\Omega$ and $V4 = 10V$.

1. (10pts) Given $V0 = 5V$, what are $V3$ and $IR2$?

   Things that are always true:

$$\begin{aligned} V0 - V1 &= I_{R1}R1 \\ I_{R1} &= I_D \\ I_D &= I_{BE} \\ I_{CE} &= I_{R2} \\ V4 - V3 &= I_{R2}R2 \end{aligned}$$

   And our diode and transistor bring certain constraints (that are also always true):

$$\begin{aligned} I_D &\geq 0 \\ I_{BE} &\geq 0 \\ gI_{BE} &\geq I_{CE} \geq 0 \end{aligned}$$

Note that we have two different non-linearities in the behavior of the circuit: one for the diode (on or off) and one for the transistor (base to emitter is on or off). This technically gives us four different cases that we would need to analyze. However, because $I_D = I_{BE}$, only two of the cases are really possible (both are on or both are off). We will therefore only consider these two cases from here out.

For $V0 = 5V$, let's assume that $I_D = I_{BE} > 0$, $V1 - V2 = V_{fD}$, and $V2 - 0 = V_{fT}$.

This implies that:

$$\begin{aligned} V2 &= 0.5V \\ V1 &= 1.5V \\ I_{R1} &= 3.5mA \end{aligned}$$

Since $I_{R1} > 0$, then we satisfy our constraint. So: the diode and transistor must be turned on.

Now, we know that:

$$350mA \geq I_{CE} \geq 0$$

Let's assume that $I_{CE} = 350mA$. This implies that:

$$\begin{aligned} I_{R2} &= 350mA \\ V3 &= -25V \end{aligned}$$

This violates the assumption (derived from the transistor) that $V3 \geq 0$. Therefore, this case cannot be true. So:

$$\begin{aligned} V3 &= 0V \\ I_{R2} &= 100mA \end{aligned}$$

2. (10pts) Given $V0 = 1.6V$, what are $V3$ and $IR2$?

   Let's assume that $I_D = I_{BE} > 0$, $V1 - V2 = V_{fD}$, and $V2 - 0 = V_{fT}$.

   This implies that:

$$\begin{aligned} V2 &= 0.5V \\ V1 &= 1.5V \\ I_{R1} &= 0.1mA \end{aligned}$$

   Since $I_{R1} > 0$, then we satisfy our constraint. So: the diode and transistor must be turned on.

   Now, we know that:

$$10mA \geq I_{CE} \geq 0$$

   Let's assume that $I_{CE} = 10mA$. This implies that:

$$\begin{aligned} I_{R2} &= 10mA \\ V3 &= 9V \end{aligned}$$

   This satisfies the assumption (derived from the transistor) that $V3 \geq 0$. So, we are done.

3. (10pts) Given $V0 = 1V$, what are $V3$ and $IR2$?

   Let's assume that $I_D = I_{BE} > 0$, $V1 - V2 = V_{fD}$, and $V2 - 0 = V_{fT}$.

   This implies that:

$$\begin{aligned} V2 &= 0.5V \\ V1 &= 1.5V \\ I_{R1} &= -0.5mA \end{aligned}$$

Since $I_{R1} < 0$, then we do not satisfy our constraint. So: this case is not correct. Therefore, the diodes and transistor must be turned off.

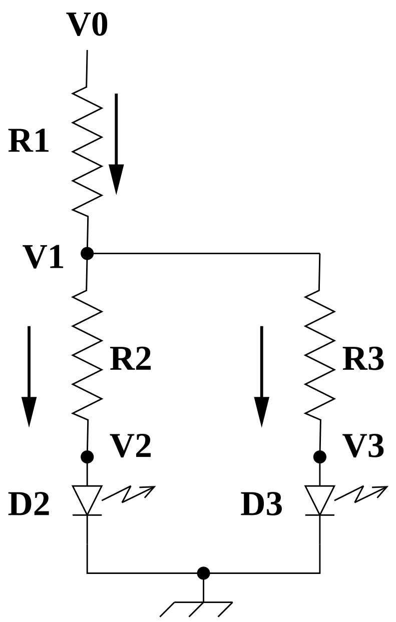
(Note: if you make the other assumption, that the diode and transistor are turned off and follow the implications, you cannot solve for $V2$. This is because $V2$ is floating – i.e., it is as if it is not connected to anything. In this case, you will not be able to explicitly show a contradiction. However, as you can see above, you can show that the opposite assumption does yield a contradiction.).

Finally:

$$\begin{aligned} I_{CE} &= 0A \\ I_{R2} &= 0A \\ V3 &= 10V \end{aligned}$$

Question 4

Consider the following circuit:



Assume $R2 = R3 = 0$, $R1 = 500\Omega$, $V_{fD2} = 1V$ and $V_{fD3} = 2V$.

1. (10pts) Assume $V_0 = 0.8V$, what are $ID2$ and $ID3$? Show your derivation.

   The following are **always** true:

$$\begin{aligned} V0 - V1 &= I_{R1}R1 \\ V1 - V2 &= I_{R2}R2 \\ V1 - V3 &= I_{R3}R3 \\ I_{R1} &= I_{R2} + I_{R3} \end{aligned}$$

   As a result of assuming $R2 = R3 = 0$, we have:

$$\begin{aligned} V1 &= V2 \\ V1 &= V3 \end{aligned}$$

   Each of our diodes can either conduct current or not. So: we have 4 possible cases.

   Case 1: Both are on.

   Specifically:

$$\begin{aligned} I_{D2} &> 0 \\ I_{D3} &> 0 \\ V2 - 0 &= 1V \\ V3 - 0 &= 2V \end{aligned}$$

   However, $V2 = V3$. So, we have a contradiction. This case cannot be true. (This will always be the conclusion for this case.)

Case 2: D2 is on; D3 is off.

$$\begin{aligned} I_{D2} &> 0 \\ I_{D3} &= 0 \\ V2 - 0 &= 1V \\ V3 - 0 &< 2V \end{aligned}$$

Since $1V < 2V$, we do not have a contradiction.

$$\begin{aligned} V1 &= 1 \\ I_{R1} &= -0.4mA \\ I_{D2} &= -0.4mA \end{aligned}$$

This is a contradiction. This case cannot be true.

Case 3: D2 is off; D3 is on.

$$\begin{aligned} I_{D2} &= 0 \\ I_{D3} &> 0 \\ V2 - 0 &< 1V \\ V3 - 0 &= 2V \end{aligned}$$

Since $2V < 1V$ is not true, then we have a contradiction. (Note that this will always be the conclusion for this case.)

Case 4: D2 is off; D3 is off.

$$\begin{aligned} I_{D2} &= 0 \\ I_{D3} &= 0 \\ V2 - 0 &< 1V \\ V3 - 0 &< 2V \end{aligned}$$

Therefore:

$$
\begin{aligned}
I_{R2} &= 0 \\
I_{R3} &= 0 \\
I_{R1} &= 0 \\
V1 &= 0.8V
\end{aligned}
$$

This is consistent with our conclusions about $V2$ and $V3$. So, we are done (and this is the answer).

2. (10pts) Assume $V_0 = 1.5V$, what are $ID2$ and $ID3$? Show your derivation.

   Case 2: D2 is on; D3 is off.

$$
\begin{aligned}
I_{D2} &> 0 \\
I_{D3} &= 0 \\
V2 - 0 &= 1V \\
V3 - 0 &< 2V
\end{aligned}
$$

   Therefore:

$$
\begin{aligned}
V1 &= 1 \\
I_{R1} &= 1mA \\
I_{D2} &= 1mA
\end{aligned}
$$

   This is not a contradiction. So, we are done.

   For giggles, let's assume: **Case 4: D2 is off; D3 is off.**

$$
\begin{aligned}
I_{D2} &= 0 \\
I_{D3} &= 0 \\
V2 - 0 &< 1V \\
V3 - 0 &< 2V
\end{aligned}
$$

Therefore:

$$\begin{aligned} I_{R2} &= 0 \\ I_{R3} &= 0 \\ I_{R1} &= 0 \\ V1 &= 1.5V \end{aligned}$$

This contradicts our constraint that $V2 < 1$. So, this case cannot be true.

3. (10pts) Assume $V_0 = 2.5V$, what are $ID2$ and $ID3$? Show your derivation.

   Case 2: D2 is on; D3 is off.

$$\begin{aligned} I_{D2} &> 0 \\ I_{D3} &= 0 \\ V2 - 0 &= 1V \\ V3 - 0 &< 2V \end{aligned}$$

Therefore:

$$\begin{aligned} V1 &= 1 \\ I_{R1} &= 3mA \\ I_{D2} &= 3mA \end{aligned}$$

This is not a contradiction. So, we are done.

Question 5 (GRADUATE ONLY)

Consider the circuit from Question 4.

Assume $R2 = 300\Omega$ and $R3 = 400\Omega$.

1. (20pts) Show $V1$, $ID2$ and $ID3$ as a function of $V0$.

   First we will focus on the general case (arbitrary $R1$ and $R2$). Since we are going to vary $V0$, we are going to examine each of the four cases and figure out the range of $V0$ for which they are not contradictory.

   First of all, an implication of what is always true (from question 4):

$$\frac{V0 - V1}{R1} = \frac{V1 - V2}{R2} + \frac{V1 - V3}{R3} \tag{1}$$

   Case 1: Both are on.

   Specifically:

$$\begin{aligned} I_{D2} &> 0 \\ I_{D3} &> 0 \\ V2 - 0 &= 1V \\ V3 - 0 &= 2V \end{aligned}$$

   Therefore:

$$\begin{aligned} I_{R2} &> 0 \\ I_{R3} &> 0 \\ I_{R1} &> 0 \end{aligned}$$

   These can only be the case if:

$$\begin{aligned} V0 &> V1 \\ V1 &> V2 = 1V \\ V1 &> V3 = 2V \end{aligned}$$

So: $V1 > 2V$.

Solving equation 1 for $V1$, we get:

$$V1 = \frac{R2R3V0 + R1R3V2 + R1R2V3}{R2R3 + R1R3 + R1R2}$$

Combining this with the inequality and solving for $V0$ gives us:

$$V0 > \frac{R1}{R2} + 2V$$

Finally:

$$\begin{aligned} I_{D2} &= I_{R2} = \frac{V1 - V2}{R2} \\ I_{D3} &= I_{R3} = \frac{V1 - V3}{R3} \end{aligned}$$

Case 2: D2 is on; D3 is off.

$$\begin{aligned} I_{D2} &> 0 \\ I_{D3} &= 0 \\ V2 - 0 &= 1V \\ V3 - 0 &< 2V \end{aligned}$$

So:

$$\begin{aligned} I_{D2} &= I_{R2} = I_{R1} \\ V1 &= V3 \\ V1 &< 2V \end{aligned}$$

$$\begin{aligned} \frac{V0 - V1}{R1} &= \frac{V1 - V2}{R2} \\ V1 &= \frac{R2V0 + R1V2}{R2 + R1} \end{aligned}$$

Combining this last equation with the inequality and solving for $V0$, gives us:

$$V0 \quad < \quad \frac{R1}{R2} + 2V \tag{2}$$

In order for positive current to flow from $V0$ to $V2$ through the two resistors, $V0 > V2$. Therefore $V0 > 1V$.

And:

$$I_{D2} \quad = \quad I_{R2} = \frac{V1 - V2}{R2}$$

Case 3: D2 is off; D3 is on.

$$\begin{aligned} I_{D2} &= 0 \\ I_{D3} &> 0 \\ V2 - 0 &< 1V \\ V3 - 0 &= 2V \end{aligned}$$

Since $2V < 1V$ is not true, then we have a contradiction. (Note that this will always be the conclusion for this case.)

Case 4: D2 is off; D3 is off.

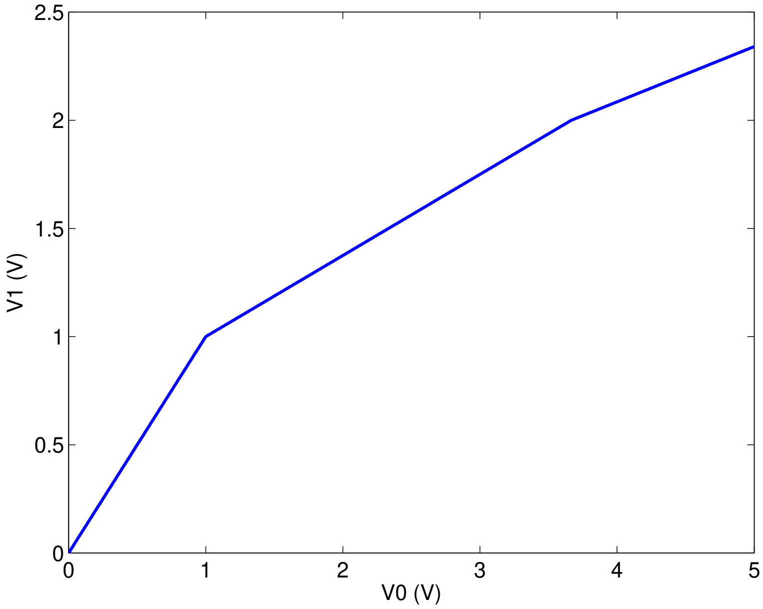
$$\begin{aligned} I_{D2} &= 0A \\ I_{D3} &= 0A \\ V2 - 0 &< 1V \\ V3 - 0 &< 2V \end{aligned}$$

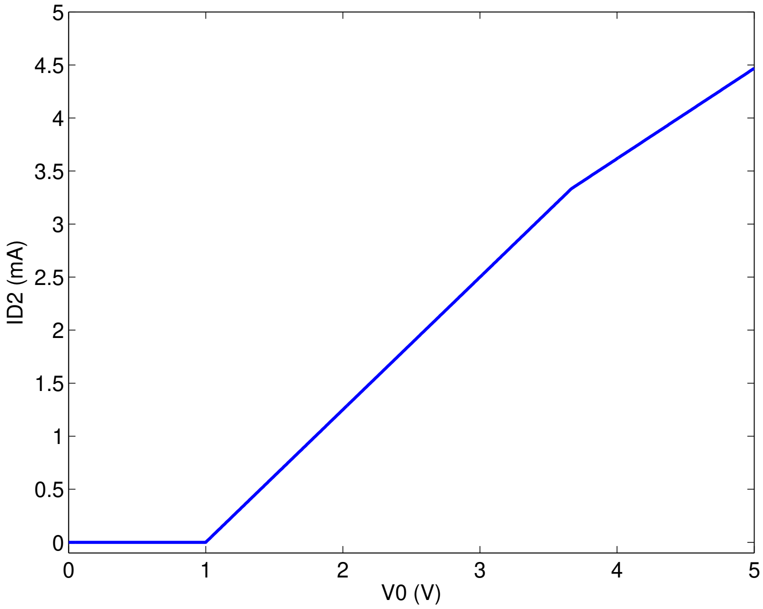
Therefore:

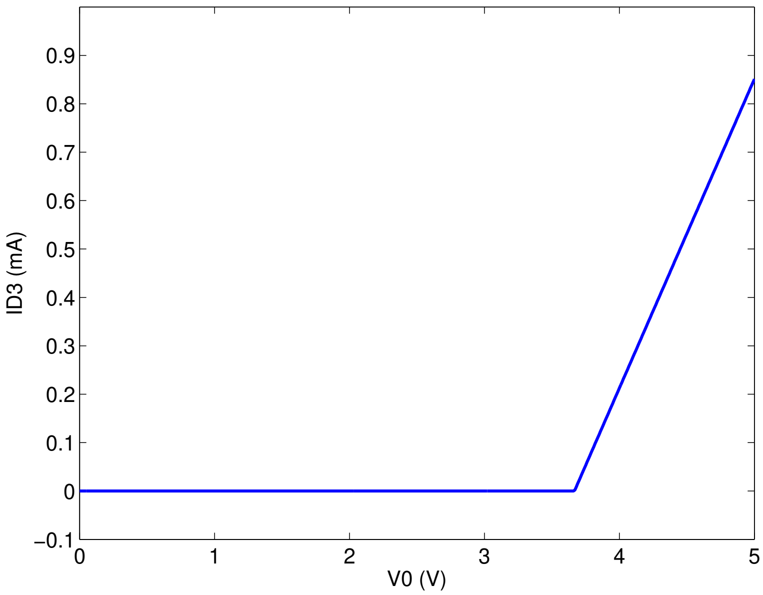
$$
\begin{aligned}
I_{R1} &= 0A \\
V1 &= V0 = V2 = V3
\end{aligned}
$$

This can only be true when $V0 < 1V$.

And now for the plots:




ID2 (mA)
V0 (V)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
1
2
3
4
5


ID3 (mA)
V0 (V)
−0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0
1
2
3
4
5